Evidence-based Practices for Teaching Mathematics to Students with Math Difficulties (MD)

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2009 and Mathematics

- Math reform and high-stakes tests
- Inclusion of students with learning, attention, and behavior disorders in general education classrooms

Prevalence of MD

- Between 5% and 8% of children have MD.
- Children with MD tend to fall further and further behind as they go through school.
  - Multiple topics (geometry, measurement)
  - Multiple concepts (place value, proportion)
  - Specialized vocabulary (equation, square root)
  - Multiple skills (whole numbers, fractions)

Three Questions

- Why do some students have so much difficulty learning mathematics?
- What do we know about evidence-based practices in mathematics?
- What are the challenges in delivering mathematics instruction in inclusive classrooms?

Why do some students have so much difficulty learning mathematics?

- Cognitive characteristics

- Behavioral characteristics

They…

can’t (ability)

won’t (oppositional) or

don’t (forget, distracted, etc.)

Cognitive Characteristics

- Semantic memory problems
  – Math fact retrieval (memorizing multiplication facts)
- Procedural problems
  – Retaining information in working memory
  – Monitoring counting processes
  – Regrouping
- Visuo-spatial problems
  – Representing numerical relationships
  – Difficulties in measurement, place value, geometry, aligning numbers.

Cognitive Characteristics

MATH WORD PROBLEM SOLVING

- Problem representation difficulties
  - Translating linguistic information
  - Transforming numerical information
- Problem execution difficulties
  - Developing a logical plan
  - Estimating the answer
  - Computing the answer
  - Checking the solution

Cognitive Characteristics

- Self-regulation problems

  Cannot locate and correct errors….

  – Self-instruction difficulties
  – Self-questioning difficulties
  – Self-monitoring difficulties

- Generalization problems
  – Across settings (home, school)
  – Across tasks (class assignments, homework)

Behavioral/ Characteristics

- Attention problems

  (careless, distractible, disorganized, difficulty listening and following directions, off-task)
- Impulsivity

  (self-control, acting without thinking, fidgety, off-task, completing work)
- Motivation and self-efficacy problems

What do we know about evidence-based practices in mathematics?

- Two best approaches for teaching students with MD
  - Direct Instruction (DI - basic skills instruction)
  - Cognitive Strategy Instruction (CSI - problem solving instruction)

DI and CSI Similarities

- Similar instructional procedures (highly structured and fast-paced approaches)
  - Modeling
  - Verbal rehearsal
  - Cueing and prompting
  - Corrective feedback
  - Reinforcement
  - Guided practice
  - Independent practice
  - Distributed practice
  - Mastery learning

DI and CSI Differences

- DI
  - didactic, teacher-directed
  - scripted lessons
  - focus on recall and basic skills
- CSI
  - interactive and explicit
  - focus on processes and problem solving
  - process or cognitive modeling (demonstrating out loud how successful problem solvers think and behave
  - self-regulation training

DI Research in Math

- Drill and practice to improve math fact recall and computation skills
  - –Declarative knowledge (math facts)
  - –Procedural knowledge (algorithms)
- Repetition and practice

DI Research in Math

- Used frequently with manipulatives (concrete materials…blocks, sticks, etc.)
- Example: Funkhouser (1995) K and grade 1
  - Candies in squares to represent numbers 1 to 5 (counting)
  - Change configurations of numbers
  - Introduce the plus (+) sign
  - Add squares
  - Introduce vertical and horizontal basic addition facts
  - Use drill and practice to memorize facts

DI Research in Math

- Examples: Burns (2005), Cooke & Reichard (1996) for elementary school students
  - Incremental rehearsal strategy (10 facts)
  - Drill and practice
  - Gradually increasing ratio of known to unknown multiplication facts
  - Flashcards and piles

DI Research in Math

- Combined with self-monitoring (e.g., Levendoski & Cartledge, 2000)
  - Self-monitoring checklists
  - Teacher cueing
  - Self-cueing

Self-Monitoring Checklist Subtraction with Regrouping

Did I?		
Did I copy the problem correctly?	Yes	No
Did I regroup correctly?	Yes	No
Did I borrow correctly?	Yes	No
Did I subtract all the numbers?	Yes	No
Did I check my answer to make sure it is correct?	Yes	No

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Combined with peer-assisted learning (e.g., Owen and Fuchs, 2002)

- Same-age peers (high achieving with low achieving)
- Cross-age peers (about 2 years difference)
- Students become tutors
- Students change places (tutor, tutee)

Steps for Finding Half

Finding Half

Jenny has 8 oranges. She wants to give ½ of the oranges to her friend. How many oranges will she have left?

STEPS:

1. Read the problem.
2. Draw circles to show the number.
3. Draw a box with a line down the middle.
4. Cross out a circle and draw it in the left box. Cross out the next circle and put it in the right box. Do that for all of the circles. If you have a left-over circle, draw a line through the circle and draw half a circle in each box.
5. Count the circles in the left box and the right box to make sure you have the same number in each box.
6. How many circles are in each box? Write the number.

DI Research in Math

Concrete-Representational-Abstract (CRA) sequence and DI (e.g., Witzel, Mercer, & Miller, 2003)

- Concrete (using 3-dimensional objects to represent the problem)
- Representational (using visual representations of the problem)
- Abstract (using number symbols to represent the problem)

DI Research in Math

CRA teaching sequence (4 steps)

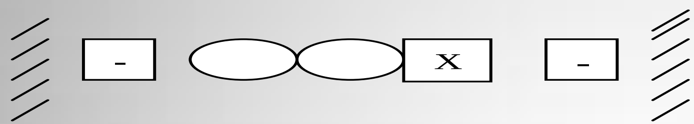
- Introduce the lesson
- Model the new procedure
- Guide students through the procedure
- Move students toward working independently

CRA Teaching Sequence for Reducing Algebraic Expressions

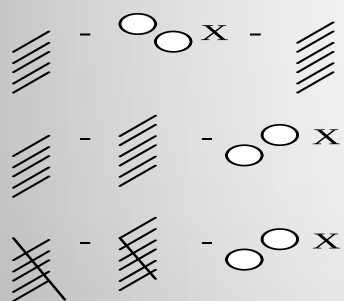
Figure 1

Concrete, Representational, and Abstract Examples of a Reducing Expressions Problem

Step 1: A **concrete** representation of 5 – 2X – 6 must use manipulative objects. For this problem it would appear in this order: five small sticks, a minus sign, one coefficient marker, an X, a plus symbol, a large stick, an equal line, and three small sticks. Manipulating the objects leaves the answer as a minus sign with one stick remaining and a minus sign followed by two cups of X.



Step 2: A pictorial **representation** would closely resemble the concrete objects but could be drawn exactly as it appears here. A student solves representational problems exactly as she would solve them concretely. For example,



The student arranges the lines together and the coefficients and unknowns separately.

The student crosses out an equal number of lines from each set with opposite signs. This leaves one line with a minus in front.

The answer remains  or -1-2X

Step 3: An **abstract** problem is written using Arabic symbols as displayed in most textbooks and standardized exams. Students in the comparison group used this format for problem solving during each lesson. The Multisensory group only used this format after concrete and representational manipulation. To solve abstract problems, students write each step to solving this problem. For example,

$5 - 2X - 6$
$^{+}5 - 6 - 2X$
$^{-}1 - 2X$

CSI Research in Math

- Most CSI studies focus on math word problem solving.
- All studies incorporate self-regulation training.
- Cognitive routines for solving math problems.
- Four examples:

CSI Research in Math

Cassel & Reid (1996)

- Four students (LD and MR)
- Addition/subtraction problems
- Self-Regulated Strategy Development (SRSD) (Harris & Graham, 1993)

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STRATEGY STEPS

- Read the problem out loud.
- Find and highlight the question and write the label.
- Ask what are the parts of the problem and circle the numbers needed.
- Set up the problem by writing and labeling the numbers.
- Reread the problem and tie down the sign.
- Discover the sign – re-check the operation.
- Read the number problem.
- Answer the number problem.
- Write the answer and check by asking if the answer makes sense.

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SELF-REGULATION PROMPTS

- Problem definition (e.g.,What is it I have to do?)
- Planning (e.g.,How can I solve this problem?)
- Strategy use (e.g., The strategy will help me organize my problem solving and remember all the things I need to do.)
- Self-monitor (e.g., To help me remember what I have done, I can check off the steps of the strategy as they are completed.)
- Self-evaluation (e.g., How am I doing? Did I complete all the steps?)
- Self-reinforcement (e.g., Great, I’m half way through the strategy.)

CSI Research in Math

Case, Harris, & Graham (1992)

- Four students with MD
- Grades 5 and 6
- Addition and subtraction problems
- SRSD instruction
  - Introduce the strategy
  - Model the procedures
  - Memorize the strategy
  - Guided practice
  - Corrective feedback and practice
  - Independent practice

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STRATEGY STEPS

- Read the problem out loud.
- Look for important words and circle them.
- Draw pictures to tell what is happening.
- Write down the math sentence.
- Write down the answer.

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SELF-REGULATION PROMPTS

- Problem definition (e.g.,What is it I have to do?)
- Planning (e.g., How can I solve this problem?)
- Strategy use (e.g.,The five-step strategy will help me look for important words.
- Self-evaluation (e.g., How am I doing?)
- Self-reinforcement (e.g., I did a nice job.)

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Hutchinson (1993)

- 12 adolescents with MD
- Algebra problem solving
- Prompt cards for problem representation
- Structured worksheets

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3 types of problems :

- Relational problems (Eddie walks 6 miles farther than Amelia. If the total distance walked by both is 32 miles, how far did each walk?)
- Proportion problems (On a map a distance of 2 inches represents 120 miles. What distance is represented on this map by 5 inches?)
- Two-variable, two-equation problems (Sam traveled 760 miles, some at 80 miles per hour and some at 60 miles per hour. The total time taken was 8 hours. Find the distance Sam traveled at 80 miles per hour.)

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Self-Questions for Representing Algebra Word Problems

- Have I read and understood each sentence? Are there any words whose meaning I have to ask.
- Have I got the whole picture, a representation, for the problem?
- Have I written down my representation on the worksheet? (goal, unknown(s), known(s), type of problem, equation
- What should I look for in a new problem to see it is the same kind of problem?

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Self-Questions for Solving Algebra Word Problems

- Have I written an equation?
- Have I expanded the terms?
- Have I written out the steps of my solution on the worksheet? (collected like terms, isolated unknown(s), solved for unknown(s), checked my answer with the goal, highlighted my answer)
- What should I look for in a new problem to see if it is the same kind of problem?

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STRUCTURED WORKSHEET

- Goal: ____
-
-
- What I don't know:-----------------------____
-
- What I know:
-
-
- I can write/say this problem in my own words. Draw a picture.
-
-
-
-
- Kind of problem:____
-
- Equation:
-
-
-
-
- Solving the equation:
-
-
-
-
- Solution:
-
-
- Compare to goal:
-
-
- Check:
-

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Montague (1992); Montague et al. (1993), Montague, Enders, & Dietz (2009)

- *Solve It!* – the cognitive routine
- Middle school and high school students
- Students with MD and average-achieving students
- Large research project in Miami, Florida (34 schools)

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SOLVE IT! - COGNITIVE PROCESSES

Reading the problem

(reading, rereading, identifying relevant/irrelevant information).

Paraphrasing

(translating the linguistic information by putting the problem into one's own words without changing the meaning of the "story" or "situation").

SOLVE IT! - COGNITIVE PROCESSES

Visualizing

(transforming the linguistic and numerical information to form internal representations in memory through a drawing or image that shows the relationships among the components of a problem).

SOLVE IT! - COGNITIVE PROCESSES

Hypothesizing about problem solutions (establishing a goal, looking toward the outcome, and setting up a plan to solve the problem by deciding on the operations that are needed, selecting and ordering the operations, and transforming the information into correct equations and algorithms).

SOLVE IT! - COGNITIVE PROCESSES

Estimating the outcome or answer

(validating the process as well as the product by predicting the outcome based on the question/goal and the information presented).

SOLVE IT! - COGNITIVE PROCESSES

Computing the outcome or answer

(recalling the correct procedures for the basic operations needed for solution – calculator skills are taught/reinforced here).

Checking

(students become aware of problem solving as a recursive activity and learn how to check both process and product by checking their understanding and representation as well as the accuracy of the process, procedures, and computation).

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SELF-REGULATION STRATEGIES

SAY (Self-instructions)

ASK (Self-questioning)

CHECK (Self-monitoring)

Solve It! – CSI Instruction

READ (for understanding)

Say: Read the problem. If I don't understand, read it again.

Ask: Have I read and understood the problem?

Check: For understanding as I solve the problem.

PARAPHRASE (your own words)

Say: Underline the important information. Put the problem in my own words.

Ask: Have I underlined the important information? What is the question? What am I looking for?

Check: That the information goes with the question.

VISUALIZE (a picture or a diagram)

Say: Make a drawing or a diagram. Show the relationships among the problem parts.

Ask: Does the picture fit the problem? Did I show the relationships?

Check: The picture against the problem information.

HYPOTHESIZE (a plan to solve the problem)

Say: Decide how many steps and operations are needed. Write the operation symbols (+, -, x, and /).

Ask: If I …, what will I get? If I …, then what do I need to do next? How many steps are needed?

Check: That the plan makes sense.

ESTIMATE (predict the answer)

Say: Round the numbers, do the problem in my head, and write the estimate.

Ask: Did I round up and down? Did I write the estimate?

Check: That I used the important information.

COMPUTE (do the arithmetic)

Say: Do the operations in the right order.

Ask: How does my answer compare with my estimate? Does my answer make sense? Are the decimals or money signs in the right places?

Check: That all the operations were done in the right order.

CHECK (make sure everything is right)

Say: Check the plan to make sure it is right. Check the computation.

Ask: Have I checked every step? Have I checked the computation? Is my answer right?

Check: That everything is right. If not, go back. Ask for help if I need it.

Conclusions

- Principles and practices of DI and CSI are part of instructional "packages" across domains and grade levels (e.g., modeling, verbal rehearsal, guided practice, etc.)
- For evidence-based practice, several studies must establish effectiveness of the intervention.
- What works, for whom does it work, and under what conditions does it ?

What are the challenges of delivering math instruction in inclusive classrooms?

- Is specialized or differentiated instruction necessary (assessment and monitoring)?
- Can I adapt or modify instruction to meet the needs of my students?
- Do I have administrative support?
- Do I collaborate with other teachers to improve instruction?